Analysis Techniques for Nonlinear Offshore Systems under Random Excitation

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ABSTRACT

The dynamics of an offshore system is influenced by various effects which are inherently random and nonlinear in nature: These effects can result from different sources such as hydrodynamic forces, coupling of different vessels or nonlinear restoring forces of mooring systems. Therefore, results from a linear analysis such as frequency response calculations may be comparatively easy to obtain, but their validity is usually limited to small amplitude motions.

The work includes models of floating vessels at different levels of simplification, from a single moored barge to systems with multiple components such as crane operations.

In order to determine or at least approximate probability density functions, numerical techniques such as Monte Carlo simulations and statistical linearizations are addressed as well as analytical methods on the basis of perturbation approaches. While the analysis techniques have their specific advantages, the results agree quite well. They allow for an estimation of the probability of occurrence of extreme events in the dynamics.

1.0 INTRODUCTION

In an environment characterized by random wave and wind force, accidents resulting from the dynamical response of floating vessels are a potential danger. Accessing the probabilities of large amplitude motions or collisions mathematically is a difficult task. It not only requires a detailed description of the mechanical problem but also results in a high computational effort. The aim of the current research is to develop techniques that allow for an assessment of probabilities for extreme events and the associated expected times.

The first step in the investigation is the development of a suitable model. Mathematical descriptions of offshore systems show a wide variety – from relatively simple one or two degree-of-freedom models to large-scale multibody systems and discretized descriptions of the fluid-structure interaction or flexible components, [2].

The modeling process is usually a trade-off between complex and simple formulations: While the former give a more precise description of the mechanical interrelation of different components, the later are significantly easier to evaluate for multiple sets of parameter values or initial conditions at practicable computational costs.

A systematic evaluation of the equations of motion requires both, a precise model which is yet simple enough to evaluate numerically. It is therefore important to treat the modeling process as an integral part of the investigation: Different techniques for the analysis require specific formulations of the equations of motion and a large number of different approaches have been presented in literature, see e.g. [1] or [4] and the references therein.

2.0 MODELING ASPECTS

In recent years a number of studies about the nonlinear dynamics of marine systems have focused on the deterministic aspects of the responses. Excitations resulting from ocean waves are often idealized as harmonic functions. The coexistence of multiple steady-state oscillations with different amplitudes in certain parameter ranges is a well-known phenomenon for such systems. These different attractors also include subharmonic or chaotic responses. Since the observed dynamics in these cases is highly dependent on the initial conditions, the question about the robustness of the different solutions arises. Uncertainties with respect to the state of the system, its parameters or the external excitation could cause a completely different dynamical response and are therefore highly critical.

2.1 Floating Systems

When describing the dynamics of a floating system, part of the equations of motion is due to the surrounding fluid, part of it results from the internal structure: Mechanical coupling of different rigid bodies or the deformation of flexible components leads to nonlinear terms in the equations of motion. The example considered here refers to the motion of a moored floating barge. The restoring force of the mooring system is due to the weight of a catenary system. A horizontal displacement of the vessel changes the catenary curves of the mooring lines. Parts of the heavy chains are lifted from the ground on one side of the vessel and are lowered to the ground on the other side. This effect contributes a nonlinear restoring force which for one individual chain is commonly described by a hyperbolic cosine function.

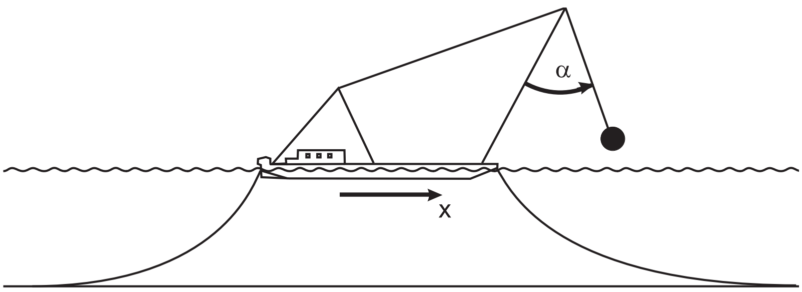

Figure 1 : Model of a floating crane

Taking into account the effect of several chains it is more efficient to approximate the resulting total restoring force by a polynomial form

$$f_m = -c_1 X - c_2 X|X| - c_3 X^3 \quad (1)$$

Assuming that the fluid-structure interaction can be described by a linear model, the equation of motion takes the form

$$m\ddot{X} = q(X, \dot{X}, t) + f_e(t) \quad (2)$$

In this case of a one degree-of-freedom model the wave excitation f_e only includes the X-component of $\mathbf{f}_e$. The total mass m includes the mass of the barge m_p and the added mass a due to the motion of the fluid. External forces including viscous drag

$$f_{vd} = -\frac{1}{2}\rho c_D BT|\dot{X}|\dot{X} \quad (3)$$

radiation damping

$$f_{rd} = -b_x \dot{X} \quad (4)$$

and mooring line forces are combined in q. The parameters in these equations are the density of water ρ, a drag coefficient c_D, the width B, and draft T of the vessel, and the added damping coefficient b_x.

As a second example we take a similar barge but also include the motion of a swinging load, which is suspended from a crane, see [3]. The equations of motion then take the form

$$\mathbf{M}(\mathbf{Y})\ddot{\mathbf{Y}} + \mathbf{k}(\mathbf{Y},\dot{\mathbf{Y}}) - \mathbf{q}(\mathbf{Y},\dot{\mathbf{Y}}) = \mathbf{f}_e(t) \quad (5)$$

with the total mass matrix

$$\mathbf{M} = \begin{bmatrix} m_p + m_l + a_x & l\, m_l \cos\alpha \\ l\, m_l \cos\alpha & l^2 m_l \end{bmatrix} \quad (6)$$

the vector of Coriolis and gyroscopic forces

$$\mathbf{k} = \begin{bmatrix} l m_l \dot{\alpha}^2 \sin\alpha \\ 0 \end{bmatrix} \quad (7)$$

and the vector of external forces

$$\mathbf{q} = \begin{bmatrix} -0.5BTc_d \rho \dot{X}|\dot{X}| - b_x\dot{X} - c_1 X - c_2 X|X| - c_3 X^3 \\ -g l m_l \sin\alpha \end{bmatrix} \quad (8)$$

Herein the mass of the load is denoted by m_l, the length of the hoisting rope is l, the swing angle is α and g is the acceleration due to gravity. For more details on the modeling of the system please refer to [3].

In order to get different levels of idealization for the wave forces, we first assume that the forcing takes the form of a harmonic function. This is also referred to as the undisturbed case. Then, in the next step an underlying harmonic function is perturbed by an additive disturbance. This gives the superposition of a harmonic force at the dominant frequency and a small random component given by the output of a filter, which in the simplest case would constitute a series of random impulses

$$f_e(t) = A(k_r \cos(\Omega t) + k_i \sin(\Omega t)) + A^2 p_d + \varepsilon\xi \quad (9)$$

where ξ is random white noise and the magnitude of the disturbance is controlled by the factor ε. The disturbances studied in this paper are random disturbances only.

3.0 ANALYSIS

Several techniques may be used in order to approximate the dynamical response of a randomly excited system. By far the most widely used are Monte Carlo techniques. Since these techniques are computationally costly, we here propose two different techniques, an analytical approximation and a numerical procedure, which may serve to characterize the dynamics.

3.1 Analytical Technique

There are very few techniques which allow for an analytical treatment of nonlinear stochastic systems. Some of the few approaches which give at least a little insight in a system's dynamics are perturbation techniques. If a problem can be formulated in such a way that it can be considered as a perturbation of some other system with a known solution, one can often express the effect of the perturbation in terms of a series expansion

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \ldots \qquad (10)$$

These techniques are widely used for deterministic systems.

For the randomly perturbed systems treated here, we use the multiple scales technique. As the excitation is considered to be composed out of a strong harmonic component and a smaller random term, the resulting response is assumed to be a perturbation of a deterministic system.

For the case of deterministic forcing, the system considered here has been investigated in [3]. The effects which have been investigated there included different primary and subharmonic resonances. Here we add a small random perturbation to the forcing term and include it in the analysis which otherwise is very similar to the deterministic case.

We start by sorting the terms in the equations of motion in such a way that the linear terms or displacement and acceleration remain on the left hand side and all other components form the right hand side:

$$\overline{\mathbf{M}}\ddot{\mathbf{y}} + \overline{\mathbf{K}}\mathbf{y} = -\mathbf{B}\dot{\mathbf{y}} + \mathbf{g}(\mathbf{y}, \dot{\mathbf{y}}, \ddot{\mathbf{y}}, \chi) + \mathbf{f}_w(\Omega, t). \qquad (11)$$

The different component of the motion are then decoupled by the transformation

$$\mathbf{y} = \mathbf{\Phi}\xi . \qquad (12)$$

The matrix $\mathbf{\Phi}$ is the modal matrix of the linearized equation given in (15). This approach yields the normal form of the system

$$\ddot{\xi} + \mathbf{\Omega}_0 \xi = \mathbf{h}(\Omega t, \xi, \dot{\xi}, \ddot{\xi}, \chi). \qquad (13)$$

In contrast to the deterministic case, we include the random components χ_i in the normal form of the equations of motion. All these components are assumed to have zero-mean

$$E[\chi_i] = 0 . \qquad (14)$$

These random perturbations are added to the harmonic excitation with the frequency Ω and scaled accordingly. More detailed assumptions concerning the perturbations are added in a later step of the analysis.

We follow the procedure of the multiple scales analysis and assume that the solution of (12) takes the form

$$\xi = \xi_0(T_0, T_1, T_2) + \varepsilon \xi_1(T_0, T_1, T_2) + \varepsilon^2 \xi_2(T_0, T_1, T_2) + \dots \quad (15)$$

The terms T_i mark the different time scales with

$$T_0 = t, \quad T_1 = \varepsilon t, \quad T_2 = \varepsilon^2 t \,. \quad (16)$$

Then, sorting the different orders of ε and requiring that (17) is fulfilled for each order of ε separately, the procedure leads to a set of differential equations

$$\varepsilon^0 \quad \rightarrow \quad D_0^2 \xi_0 + \mathbf{\Omega}_0^2 \xi_0 = \mathbf{0}, \quad (17)$$

$$\varepsilon^1 \quad \rightarrow \quad D_0^2 \xi_1 + \mathbf{\Omega}_0^2 \xi_1 = -2 D_0 D_1 \xi_0 + \mathbf{h}_1\left(\xi_0 D_0^2 \xi_{0,} (D_0 \xi_0)^2\right), \quad (18)$$

$$\varepsilon^2 \quad \rightarrow \quad D_0^2 \xi_2 + \mathbf{\Omega}_0^2 \xi_2 = -\left[2 D_0 D_2 + (D_1)^2\right]\xi_0 - 2 D_0 D_1 \xi_1 + \mathbf{h}_2(\Omega t, \xi_0, \xi_1, \chi). \quad (19)$$

On the right hand side of these equations only the excitation term

$$\mathbf{h}_2(\Omega t, \xi_0, \xi_1, \chi) = \mathbf{h}_2(\Omega t, \xi_0, \xi_1) + \sum_l \frac{\omega_l}{2\pi} \int_0^{\frac{2\pi}{\omega_l}} \chi \exp\{-\omega_l T_0\} dT_0 \,. \quad (20)$$

has a contribution due to the random perturbation. All other terms remain unchanged compared to the deterministic analysis and just like in the deterministic case we therefore introduce the detuning ν which describes the difference between the excitation Ω and the resonance frequency ω_j

$$\varepsilon^2 \nu = \Omega - \omega_j \,. \quad (21)$$

With this approach, the form of the first and second order approximations of the solution also remain unaffected by the random perturbation giving

$$\xi_{0j} = \frac{1}{2} a_j(T_1, T_2) \exp\{i\beta_j(T_1, T_2)\} \exp\{i\omega_j T_0\} + c.c., \quad (22)$$

and

$$\xi_1 = \sum_{j,k} \left\{ A_j A_k (T_2) \mathbf{S}_{jk}^{(1)} \exp\{i(\omega_j + \omega_k) T_0\} + A_j \overline{A}_k (T_2) \mathbf{S}_{jk}^{(2)} \exp\{i(\omega_j + \omega_k) T_0\} + c.c. \right\}, \quad (23)$$

where in both cases $c.c.$ stands for the complex conjugate terms. It is the elimination of secular terms in (23) that first leads to contributions of the random terms:

$$\frac{da_j}{dT_2} = -\mu_j a_j - d_j a_j |a_j| + f_j \sin(\gamma_j + \delta_j) + \chi_{j1}, \tag{24}$$

$$a_j \left(\nu - \frac{d\gamma_j}{dT_2} \right) = \sum_l \Lambda_{jl} a_j a_l^2 - f_j \cos(\gamma_j + \delta_j) + \chi_{j2}, \tag{25}$$

$$\frac{da_k}{dT_2} = -\mu_k a_k - d_k a_k |a_k| + \chi_{k1}, \tag{26}$$

$$a_k \frac{d\beta_k}{dT_2} = \sum_l \Lambda_{kl} a_k a_l^2 + \chi_{k2}, \tag{27}$$

The coefficients Λ_{jl} and Λ_{kl} are functions of the system's parameters. With $\phi = \Omega t - \gamma$, the terms of the random perturbations are

$$\chi_{j1} = -\frac{1}{2\pi\omega_j} \int_0^{2\pi} \chi \sin\phi \, d\phi, \tag{28}$$

$$\chi_{j2} = \frac{1}{2\pi\omega_j} \int_0^{2\pi} \chi \cos\phi \, d\phi. \tag{29}$$

The coefficients Λ_{jk}, the hydrodynamic forces f_j, the phases δ_j, and the damping terms μ_j and d_j remain the same as in the deterministic case and we therefore refer to [3] for details.

In order to determine the effect of the random perturbation we here split the amplitudes and phases into a deterministic (d) and a random (r) component,

$$a_j = a_{jd} + a_{jr}, \tag{30}$$

$$\gamma_j = \gamma_{jd} + \gamma_{jr}. \tag{31}$$

This approach leaves the deterministic part of the solution unchanged compared to the unperturbed case, since all random components are accounted for in the a_{jr} and γ_{jr} terms. Linearization of the equations for the random contributions leads to

$$\frac{da_{jr}}{dT_2} = -\mu_j a_{jr} - d_j a_{jr} |a_{jd}| + \chi_{j1}, \tag{32}$$

$$a_{jd} \left(\nu - \frac{d\gamma_{jr}}{dT_2} \right) = \sum_l \Lambda_{jl} a_{jr} a_{ld}^2 + \chi_{j2}, \tag{33}$$

$$\frac{da_{kr}}{dT_2} = -\mu_k a_{kr} - d_k a_{kr} |a_{kd}| + \chi_{k1}, \tag{34}$$

$$a_{kd}\frac{d\beta_{kr}}{dT_2}=\sum_l \Lambda_{kl}a_{kr}a_{ld}^2+\chi_{k2}, \tag{35}$$

Considering the equations for the amplitudes a_{jr} and a_{kr} in (36) and (39) we get a zero-mean contribution

$$E[a_{jr}]=0, \tag{36}$$

$$E[a_{kr}]=0, \tag{37}$$

and the autocovariances

$$E[a_{jr}(t_1),a_{jr}(t_2)]=\int_0^{t_2}\int_0^{t_1}\exp\{(-\mu_j-d_j|a_{kd}|)[(t_1-\tau_1)+(t_2-\tau_2)]\}E[\chi_{j1}(\tau_1),\chi_{j2}(\tau_2)]d\tau_1 d\tau_2, \tag{38}$$

$$E[a_{kr}(t_1),a_{kr}(t_2)]=\int_0^{t_2}\int_0^{t_1}\exp\{(-\mu_k-d_k|a_{kd}|)[(t_1-\tau_1)+(t_2-\tau_2)]\}E[\chi_{k1}(\tau_1),\chi_{k2}(\tau_2)]d\tau_1 d\tau_2. \tag{39}$$

Until now, the only assumption made on χ was that it is zero-mean. Within the limits of the approximations made in this proposed procedure, the statistical properties of the responses can be determined from (42) and (43). For example, considering whit noise with the intensity S_0, i.e. the correlation

$$R_{\chi_j}(\tau)=2\pi S_0\delta(\tau) \tag{40}$$

we get

$$E[a_{jr}(t_1),a_{jr}(t_2)]=\frac{\pi S_0}{\omega_j^2(-\mu_j-d_j|a_{jd}|)}, \tag{41}$$

$$E[a_{kr}(t_1),a_{kr}(t_2)]=\frac{\pi S_0}{\omega_k^2(-\mu_k-d_k|a_{kd}|)}. \tag{42}$$

This means that we can approximate the effect of small random perturbations from the intensity of the random excitation component and the system's parameters. For the example of white noise the result is independent of time and independent of the detuning parameter ν.

It has to be noted however, that this approach with the multiple scales analysis only considers small nonlinearities and small perturbations. Large deviations from the solution of the deterministic system might eventually lead to jumps between different solutions. Since this cannot be handled by the approach presented above, we turn to numerical techniques in the next section.

3.2 Numerical Techniques

There are different mathematical techniques in order to obtain information on probabilities or rather probability densities. The most common approaches are based on Monte-Carlo simulations: Long sample realizations are evaluated statistically to give the required information. The advantage of this method lies in its flexibility: Even large systems with multiple degrees of freedom can be analyzed with this technique, though most investigations result in massive calculations and effects of small probability are hardly accessible.

As an alternative the statistical linearization has been shown to be applicable for large random systems. The concept used in this paper is based on normal (Gaussian) distributions, which are completely characterized by their mean value μ and their variance σ^2, or for an $n-$dimensional system the vector of mean values $\boldsymbol{\mu}$ and the covariance matrix $\mathbf{C}$, respectively. The probability density of a Gaussian distribution is given by

$$p(y)=\frac{\exp\left(-\frac{1}{2}(\mathbf{y}-\boldsymbol{\mu})^T\mathbf{C}^{-1}(\mathbf{y}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^n|\mathbf{C}|}} \tag{43}$$

This formula holds for higher order dimensions.

The advantage of using normal distributions for this method lies in the invariance of its form under a linear transformation. For a randomly forced linear system the evolution of the initial condition would always keep the shape of the typical bell curve. The main idea of the linearization technique as it is applied here is also based on a simulation in time domain, but instead of following a single trajectory, the evolution of an initial distribution is determined.

While the overall dynamics of the system under investigation may include sources of nonlinear behavior, it is still sufficiently smooth in order to apply a local linearization.

As long as the standard deviation is sufficiently small, the behavior can therefore be considered linear about the mean value of the distribution. These assumptions result in a time-stepping scheme in which the distribution $\{\mu, C\}$ with the mean value μ and the covariance matrix C is mapped onto a new distribution in the next time step $t+\Delta t$

$$\{\boldsymbol{\mu},\mathbf{C}\}_t \rightarrow \{\boldsymbol{\mu},\mathbf{C}\}_{t+\Delta t} \tag{44}$$

Where the rate of change of the mean value $\boldsymbol{\mu}$ and the covariance matrix C are determined from the equations of motion by

$$\frac{d\boldsymbol{\mu}}{dt}=E\left[\frac{d\mathbf{Y}}{dt}\right] \tag{45}$$

$$\frac{d\mathbf{C}}{dt}=E\left[\frac{d\left(\mathbf{Y}\mathbf{Y}^T\right)}{dt}\right] \tag{46}$$

It has to be noted that due to the dissipation, any distribution with a small variance will eventually spread out. Thus, the linearization about the mean value will become an increasingly crude approximation. In order to maintain the local character of the linearization, the variance is required to remain below a certain value. Once this limit is reached, the distribution is split into several parts as indicated in Figure 2 and the different parts are treated individually in the subsequent time steps. The total probability density is then computed as the sum of the individual distributions. On the other hand, different distributions are combined to one single distribution in order to reduce computational time [5].

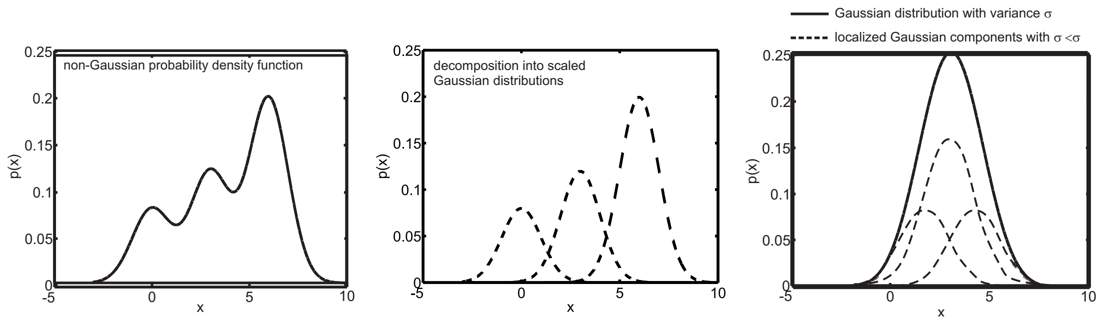

Figure 2 : Overlaying and splitting of distributions

4.0 RESULTS

Given a deterministic forcing, we first consider the floating crane without a swinging load. In this case we obtain four different steady-state solutions for the assumed operating conditions as shown in Figure 3: One period-1 type of motion and three different subharmonic responses.

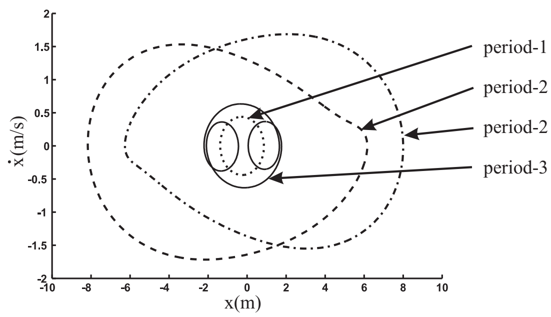

Figure 3 : Phase diagrams of the attractors of the undisturbed system.

Figure 3 indicates that the maximum displacements are significantly different for the four separate types of motion: while the period-1 and the period-3 motions have relatively moderate maximum displacements around $2m$, the two period-2 motions go up to about $8m$.

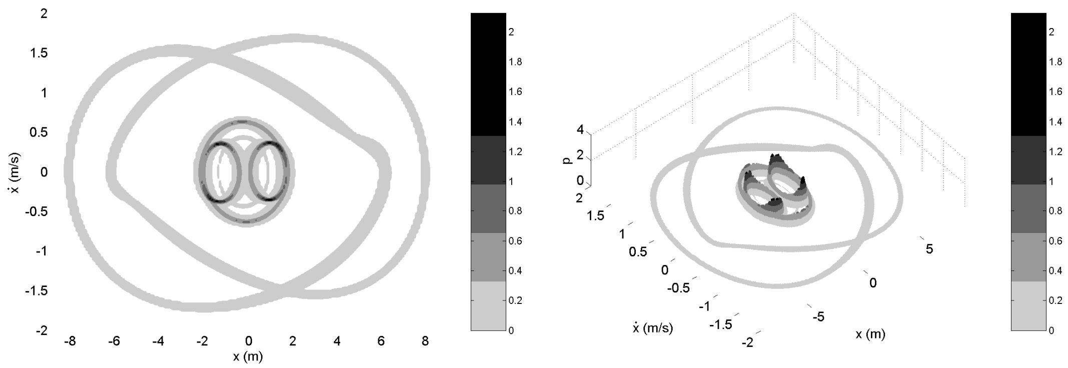

Figure 4 : Probability density functions for the disturbed system: small perturbation ($\sigma = 0.0005$).

The different types of motion – often referred to as different attractors – have been obtained by a numerical integration of the equation of motion with the same parameters but different initial conditions. All four solutions are stable: Small perturbations do not change the character of the solution, if they are sufficiently small. Such small perturbations occur naturally during the numerical integration due to truncation errors. This means that there are no jumps between the different solutions for the deterministically forced system. Only large perturbations, i.e. large changes in the velocity or the position of the barge, would cause jumps between the different solutions.

As long as the forcing is deterministic, the initial condition of the integration of motion determines which type of steady-state motion will be reached. Adding a random forcing changes this behavior significantly: Even though the system will stay close to one of the four different types of motion, random jumps between the areas surrounding these attractors are possible. In order to see this, Monte-Carlo simulations have been performed with different levels of random disturbances.

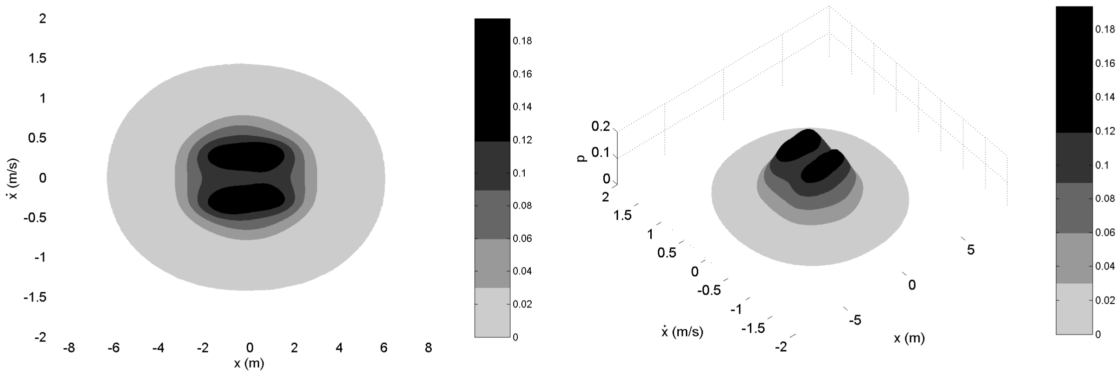

Figure 5: Probability density functions for the disturbed system: large perturbation ($\sigma = 0.01$).

The results are displayed in Figure 4 and Figure 5. It can be seen that the highest values of the probability density occur near the periodic attractors shown in Figure 3. For the small perturbation all four types of motion remain visible in the probability density function, see Figure 4. Only with higher levels of the disturbances the areas with relatively high probabilities get wider and the influence, especially of the large-amplitude attractors diminishes, Figure 5.

For the model of a crane with a swinging load, the method of statistical linearization is used in order to see the effect of uncertainties of the initial condition and random disturbances as time proceeds. Figure 6 shows the initial distribution which is assumed to be centered at the equilibrium position of an unforced system. It also shows how the initial distribution is deformed in the first three periods.

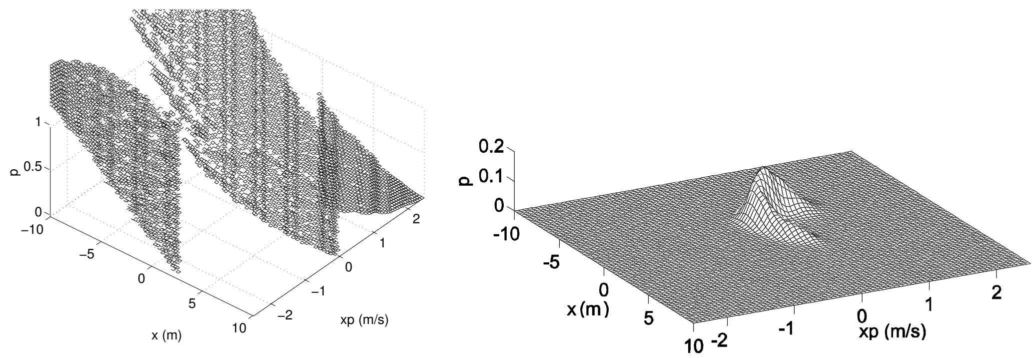

Figure 6: Initial distribution for the analysis of the floating crane and probability density after three periods of the deterministic part of the forcing

The significant reduction in the peak value and the wedge-like shape of the distribution are due to the dissipative term, caused by the random disturbance, and the nonlinearity of the system.

The fact that the period-3 motion is dominant in the deterministically forced system means that in the slightly disturbed case, the maximum of the distribution might follow either one of three different paths. These three different paths only differ by a phase change corresponding to the period T of the harmonic part of the forcing. If averaged over three periods, the distribution shows maxima near the two loops in the trajectory of the deterministic system, Figure 7.

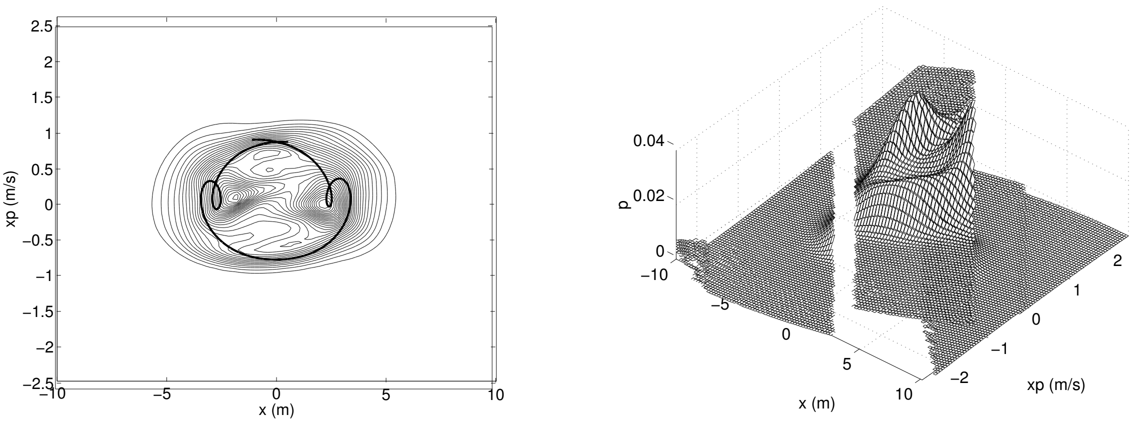

Figure 7: Contour and surface plots of the mean distribution averaged over three periods of the harmonic component of the forcing. The dark solid line marks the evolution of the corresponding deterministic system.

Even though the small uncertainties in the initial condition and the random disturbance of the forcing lead to a significant variance after only a few time steps, it should be noted that the mean follows the trajectory of the corresponding undisturbed system closely: The probability distribution averaged over the first three periods is shown in Figure 8. The peak in the middle of the surface due to the narrow initial distribution (Figure 6) is still very pronounced.

The probability densities here give information on the dynamics of a randomly disturbed system. In order to compare these results with the nearby undisturbed system the path of the maximum of the probability density in each time-step is compared to the motion of an undisturbed system. Figure 9 shows that for the crane vessel the maximum of the computed probability distribution still follows the trajectory of the corresponding deterministically forced system.

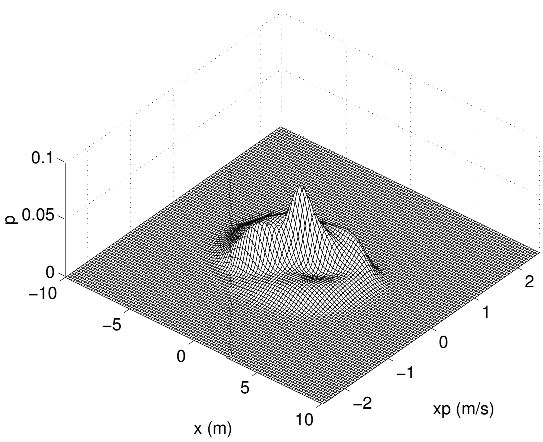

Figure 8 : Mean distribution averaged over the first three periods

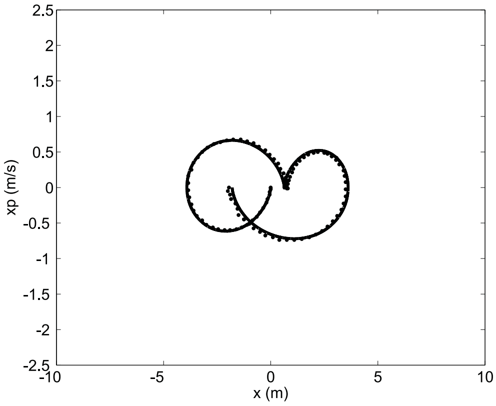

Figure 9 : Maxima of the probability density (dots) and trajectory of the deterministically forced system (solid line))

5.0 CONCLUSIONS

This paper discussed the similarities and differences between systems in regular (harmonic) and randomly perturbed waves. The system under consideration exhibits distinctly nonlinear behavior in the case of regular forcing. In addition to the simulation of the deterministic system, variations caused by random disturbances in the initial conditions as well as in the forcing have been addressed by means of the local statistical linearization.

The mathematical technique described gives insight into the robustness of results obtained from simplifying assumptions such as wave forces modeled as harmonic functions. The local statistical linearization starts with an initial condition in the form of a probability density function. The following time-stepping process is similar to the simulation for a deterministic system but instead of following a single trajectory it describes the evolution of the probability density with time.

The results reveal that the response to the idealized harmonic forcing is still close to the most probable response of the comparable system with a disturbed forcing. On the other hand, there is a small probability for the system to depart significantly from the deterministically forced system. The probability of the system to operate under potentially dangerous operating conditions, with large maximum displacements or velocities, can directly be obtained from the integration of the probability density function.

6.0 REFERENCES

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